General Certificate of Education
June 2008
Advanced Level Examination

AQA
ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM05**
Unit Mechanics 5

Wednesday 18 June 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A simple pendulum of length 2 metres is set in motion.

(a) (i) Show that the period of the motion is 2.84 seconds, correct to three significant figures. *(2 marks)*

(ii) Show that the frequency of the motion is 0.352 cycles per second, correct to three significant figures. *(1 mark)*

(b) The length of the pendulum is adjusted so that the period of its motion is 2.5 seconds. Find the adjusted length of the pendulum. *(2 marks)*

2 A particle moves in a straight line with simple harmonic motion such that its displacement at time t seconds relative to a fixed origin on this line is x metres. The motion of the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 16x = 0$$

(a) Verify that

$$x = A\cos 4t + B\sin 4t$$

where A and B are constants, is a solution to this differential equation. *(4 marks)*

(b) When $t = 0$, the particle is momentarily at rest. Show that $B = 0$. *(1 mark)*

(c) Given that $x = h\ (h > 0)$ when $t = \frac{\pi}{12}$, find A in terms of h. *(2 marks)*

(d) Find the maximum speed of the particle in terms of h. *(1 mark)*

(e) The mass of the particle is m kg. Find the magnitude of the maximum force acting on the particle during the motion. Give your answer in terms of h and m. *(2 marks)*

3 A particle P moves in a plane so that, at time t, its polar coordinates (r, θ) with respect to a fixed origin, O, are given by

$$r = t^2 \qquad \theta = \frac{9}{\pi^2}\sin\frac{\pi t}{6}$$

(a) Find the radial and transverse components of the velocity of P when $t = 3$. *(4 marks)*

(b) Find the radial and transverse components of the acceleration of P when $t = 3$. *(5 marks)*

(c) Determine the angle between the acceleration of P and OP when $t = 3$. *(2 marks)*

4 A rocket is launched from the ground so that it travels vertically upwards. The rocket ejects burnt fuel vertically downwards at a speed of $1400\,\text{m s}^{-1}$ relative to the rocket at a constant rate of $10\,\text{kg s}^{-1}$.

The initial mass of the rocket and its fuel is 1000 kg.

The velocity of the rocket at time t seconds after it is launched is $v\,\text{m s}^{-1}$.

It may be assumed that the only external force acting on the rocket is gravity. The acceleration due to gravity should be taken as constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -9.8 + \frac{1400}{100 - t}$$

(8 marks)

(b) Given that $v = 0$ when $t = 0$, show that

$$v = -9.8t + 1400\ln\left(\frac{100}{100 - t}\right)$$

(3 marks)

(c) When $t = 80$, the fuel in the rocket has all been burnt. Find the total time taken for the rocket to reach its maximum height. *(4 marks)*

5 A particle, of mass 2 kg, is suspended from a fixed point O by a light spring of natural length 0.5 metres and modulus of elasticity 49 N.

(a) Initially, the particle hangs at rest in equilibrium below O. Find the extension of the spring in this position. *(2 marks)*

(b) A force, F newtons, is then applied to the particle in a vertically downwards direction. The displacement of the particle below its equilibrium position at time t seconds later is x metres. Given that $F = 12\cos nt$, where n is a positive constant, show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 49x = 6\cos nt$$

(5 marks)

(c) In the case where $n = 5$, find an expression for x at time t. *(10 marks)*

(d) State the value of n for which resonance occurs. *(1 mark)*

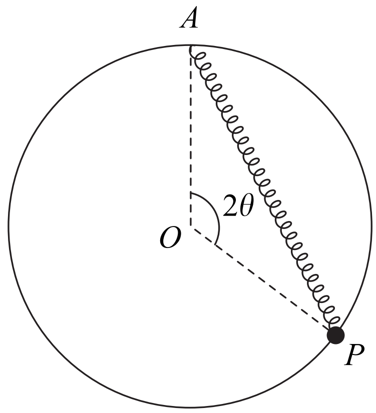
Turn over for the next question

6 A smooth circular wire, of radius a and centre O, is fixed in a vertical plane.

A small smooth bead, P, of mass m, can move freely on the wire.

The bead is attached to one end of a light spring, which has modulus of elasticity $4mg$ and natural length a. The other end of the spring is attached to A, the highest point on the wire.

The angle subtended by the spring at O is 2θ, as shown in the diagram, where $0 < \theta \leqslant \frac{\pi}{2}$.



(a) (i) Show that the elastic potential energy stored in the spring in this position is given by

$$2mga(2\sin\theta - 1)^2$$ *(3 marks)*

(ii) The gravitational potential energy is taken to be zero at the level of the lowest point on the wire. Show that the total potential energy, V, is given by

$$V = 2mga(3\sin^2\theta - 4\sin\theta + 2)$$ *(5 marks)*

(b) Find the two values of θ for which the bead is in equilibrium, giving your answers to two decimal places. *(4 marks)*

(c) Determine, for each of these values, whether the bead is in stable or unstable equilibrium. *(4 marks)*

END OF QUESTIONS